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论文题目：动力系统的复杂性与点串

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中 文 摘 要

在动力系统领域，一个重要的课题是研究与系统复杂性有关的问题。在描述系统复杂性的语言中，混沌是其中一个重要的概念。鉴于混沌现象在自然界的大量存在，关于混沌的研究不仅是现代动力系统学科最活跃的分支之一，它也成为非线性科学领域的主要课题之一。虽然近三十年来大量的研究工作揭示了许多不同类型的混沌现象及其特征，但是关于混沌的基本属性及不同混沌现象间的关联性的数学理论远未成熟。特别是两类最具代表性的混沌，Devaney 混沌和 Li-Yorke 混沌，之间的相互关系如何是一个近 20 年的公开问题。

熵是反映系统复杂性的另一个重要概念，它是由著名数学家 Kolmogorov 在 1958 年首先引入的。在熵的语言下有两类十分重要的保测系统，一类是 1958 年 Kolmogorov 引入的测度 Kolmogorov 系统，关于它的研究多年来一直是遍历理论的主要课题之一，Furstenberg、Katok、Ornstein、Rohlin、Rudolph、Sinai 和 Weiss 等著名数学家在该领域做出了许多杰出的贡献，最新的进展可参见 Rudolph 在 2002 年数学家大会 45 分钟报告。对于什么是测度 Kolmogorov 系统的拓扑对应是多年来人们一直关心的问题。1992 年法国数学家 Blanchard 在这一问题上取得了突破，成功地引入了测度 Kolmogorov 系统的拓扑对应：完全正熵系统和一致正熵系统。随后 Glasner、Host、Rudolph 和 Weiss 等众多知名学者投入到这方面的研究中，建立了熵的局部变分原理和熵对变分关系等一系列重要成果。目前，关于“拓扑 Kolmogorov 系统”仍然有许多基本的问题有待人们去解决，其中最为核心的一个问题是“拓扑 Kolmogorov 系统”的结构如何？

另一类重要的保测系统是 1967 年 Kushnirenko 引入的测度 null 系统，这类系统是熵的语言下最为简单的一类保测系统。Kushnirenko 证明了一个保测系统是 null 系统当且仅当它具有测度离散谱，因此在遍历时它相当于紧致交换群上的遍历旋转(Halmos-Von Neumann 定理)。尽管测度 null 系统的拓扑对应----拓扑 null 系统 Goodman 早在 1974 年已经引入，但直到今天对它的结构人们知道的仍然很少，这是一个相当富有挑战性的问题。

在描述系统复杂性的语言中，还有一个重要的概念就是系统的回复属性。系统回复属性的研究可追述到上世纪五六十年代 Gottschalk、Hedlund 和 Furstenberg 等人的经典工作，如 1967 年 Furstenberg 证明的“弱混合系统的回复时间集具有滤子性”。在拓扑动力中传递性、拓扑弱混合性和拓扑强混合性是三种最基本的回复属性。2000 年 Blanchard 等人在传递性和拓扑弱混合性之间成功地插入了一类回复属性：扩散性；随后，Akin、Glasner 和 Weiss 在 Fustenberg1967 年工作的基础上，使用簇的语言在特定意义下细分了弱混合性和强混合性之间的回复属性。如何有效地在三种最基本的回复属性之间插入更多有意义的回复属性进而对

系统回复属性进行更为精细的分类，这是目前人们十分关心的问题。

本文的主要目的是从遍历理论与拓扑动力系统的平行之处出发，使用局部化(点对或点串)的思想对上述提出的几个系统复杂性问题：“Devaney 混沌和 Li-Yorke 混沌的相互关系；拓扑 Kolmogorov 系统和拓扑 null 系统的结构；系统回复属性更为精细的分类”进行系统而全面地研究，以求从理论上更好地了解和把握系统的复杂性。全文共分六章，前两章是引言和预备知识。后面四章是主体内容，分为三部分，具体来讲，

第一部分将致力于解决了 Devaney 混沌是否蕴含着 Li-Yorke 混沌这一长时间的公开问题。

为此在第三章中，我们首先将空间 X 中的点对按照在作用 T 下的运动规律进行分类，讨论了这些具有特殊动力学性质的点对与混沌的关系；随后对非周期的传递系统, 我们证明了它的渐近关系和每个渐近类均为第一纲集。进而, 我们证明了如果它的 proximal 关系在对角线上某个点的一个邻域中稠密, 则它为 Li-Yorke 混沌的。作为应用, 我们得到了一个具有周期点的非周期的传递系统为 Li-Yorke 混沌的结果。由此我们解决了 Devaney 混沌是否蕴含着 Li-Yorke 混沌这一近 20 年的公开问题，对它给予了正面的回答。与此同时，我们证明了“许多”紧度量空间上存在以全空间为混沌集的动力系统, 这些空间包括一些可数的紧度量空间、康托集和任意维的连续统。这部分结果发表在 Ergodic Theory and Dynamical Systems, 21(2001), No.1, 77-91 和 Topology and its application, 117(2002), No.3, 259--272 上，引发了国内外学者对这一领域的进一步研究，已被 Akin、 Auslander、Blanchard、Glasner、Host、Kolyada、Maass、Snoha、Smital 等人 23 次引用，特别是在 Blanchard 等人证明正熵蕴含 Li-Yorke 混沌这一重要结果的论文中多处引用我们这方面的结果，Akin 和 Kolyada 在 2003 年 Nonlinearity 的文章中将我们的关于渐近关系的结果称为“Huang-Ye equivalence”。

第二部分将研究拓扑 Kolmogorov 系统和拓扑 null 系统的结构。

在第四章中, 我们研究了测度 Kolmogorov 系统的拓扑对应：n-一致正熵系统(n>1)和完全一致正熵系统。首先，我们证明了测度 Kolmogorov 系统的拓扑实现为完全一致正熵系统；接着我们使用测度 Pinsker 代数刻画了 n-一致正熵的局部化概念测度 n-熵串；然后我们证明了一个关于可测覆盖和可测剖分熵关系的定理并加强了熵的局部变分原理， 由此获得了拓扑熵串和测度熵串的变分关系。我们还使用正密度的插入集以及超空间等语言刻画了 n-一致正熵系统和完全一致正熵系统。在这一过程中，我们解决了 Blanchard、Host、Glasner 和 Weiss 提出的几个公开问题, 例如：证明了 n-一致正熵性不能蕴含(n+1)-一致正熵性。上述结果提供了解决类似问题的一般框架并且覆盖了十几年来相关问题研究中的几乎所有结果。这部分结果即将发表于 Israel Journal of Mathematics。

在第六章中, 我们对拓扑 null 系统的结构进行了研究。我们方法是首先定义序列熵的局部化概念：序列熵对，然后给出点对成为序列熵对的一些充分条件，进而结合 Ellis 半群理论证明了在拓扑动力系统中 Kushnirenko 的相应结果在忽略掉几乎一对一的扩充意义下对极小系统成立。这部分结果发表在 Ergodic Theory and Dynamical Systems,23(2003),No.5,1505-1523。与此同时，我们还使用测度 Kronecker 代数证明了测度序列熵对一定为拓扑序列熵对，然而不同于拓扑熵对与测度熵对的情形, 其逆命题并不成立, 其深刻的原因是对序列熵而言没有变分原理； 使用序列熵对的提升性质, 我们说明了每个动力系统存在最大的 null 因子系统和最大的 M-null 因子系统。这部分结果即将发表于 Annales De L’Institut Fourier。

第三部分将引入拓扑 mild-混合性等对偶性质，由此对系统回复属性进行了更为精细的分类。

在第五章中，首先我们对一个比传递性强的动力学性质引入了对偶的概念；随后我们定义了几类熟知动力学性质的对偶性质，例如拓扑 mild-混合性和几类扩散性，并使用开覆盖复杂性函数和回复时间集等多种语言去刻画这些对偶性质；最后分析了这些对偶性质与三种最基本的回复属性的相互关系，由此对系统回复属性进行了更为细致的分类。 在此过程中，我们解决了 Blanchard、Host 和 Maass 在 2000 年提出的一个公开问题：证明了 2-扩散性等价于扩散性；揭示了几类扩散系统的回复时间集是人们在组合数论、遍历理论以及拓扑群中感兴趣的重要的自然数序列，如 Poincare 序列和回复序列，因而在系统回复属性的分类中，组合数学、遍历理论、调和分析以及拓扑群的知识变为不可缺少的工具。这部分结果发表在 Nonlinearity,15(2002), No.3, 849-862 和 Ergodic Theory and Dynamical Systems, 24(2004), No.3, 825-84 上。以色列知名数学家 E.Glasner 教授最近在其综述性文章和在几所著名大学所作的学术报告中专门介绍了我们这方面的工作。

需要指出的是，我们定义的拓扑 mild-混合性可以看作遍历理论中一个重要概念测度 mild 混合性的拓扑对应，独立于我们的工作 B.Weiss 和 E.Glasner 也成功地定义和研究了拓扑 mild 混合性，他们在 B. Hasselblatt 和 A. Katok 主编的《Handbook of Dynamical systems》Volume 1B（2004 即将出版）的“On the interplay between measurable and topological dynamics”一章中，使用相当的笔墨介绍了拓扑 mild 混合性并且写到：“…We note that the definition of topological mild mixing and the above concerning this notion are new. However independently of our work Huang and Ye in a recent work also define a similar notion and give a comprehensive and systematic treatment,… ”。

在博士期间和毕业的一年内，我们还开展了以下一些与系统复杂性有关的研究：

1.部分解决了著名数学家 H.Furstenberg 在 1967 年在其经典文章提出的一个公开问题：刻画与所有极小系统不交的系统。我们指出这类系统如果是传递的，则它一定为极小点稠密的弱混合系统，我们也给出了一个系统为该类系统的充分条件，最后说明该类系统可以没有周期点。这部分结果即将发表于 Transactions of the American Mathematical Society。美国动力系统专家 E.Akin 教授最近关于我们的这一结果专门写了一个注记《Minimal maps: Notes on some results of Huang and Ye》。

2.获得了开覆盖的两种测度熵的内在关系并证明二者均具有遍历分解性，进而得到了局部的 Abramov 熵公式。这部分结果发表在 Ergodic Theory and Dynamical Systems，24(2004), No.4, 1127-1153。

3.讨论了簇混合系统的 proximal 核和几乎等度连续系统的极小集。这部分结果发表在 Nonlinearity，17(2004), No.4, 1245-1260 和 Proc. of the Steklov Inst. of Math., Vol 244(2004), 280-287。

关键词： 复杂性、点串、混沌、熵、回复属性

Complexity of Dynamical System via Tuples

Huang Wen

ABSTRACT

In the study of dynamical systems, the complexity of the system is a very important topic. To describe complexity of a system, chaos is one of the most useful concepts. Chaos is a phenomenon existing in many natural systems. It is not only one of the most active areas in dynamical systems, but also one of the main themes in nonlinear systems. Although a large amount of variant chaos and their characteristics have been revealed and studied during the past decades, the fundamental theory of chaos and connection between different types of chaos are far from mature. The relationship between two types of chaos, the Devaney chaos and Li-Yorke chaos, has been an open problem for nearly twenty years.

Entropy is another important concept in describing the complexity of a system, which is introduced by Kolmogorov in 1958 for the first time. There are two important measure-preserving systems in the language of entropy. One is the measure-theoretical Kolmogorov system, which is one of the main research topics in ergodic theory. Furstenberg, Katok, Ornstein, Rohlin, Rudolph, Sinai and Weiss did outstanding contributions in this field. Most recent developments can be found in the report by Rudolph on ICM 2002. What is an appropriate description of the topology counterpart of measure-theoretical Kolmogorov system? This problem has drawn heavy attention for decades. The first important breakthrough came at 1992. Blanchard successfully introduced the completely positive entropy (cpe) and uniformly positive entropy (upe). Later on Glasner, Host, Rudolph and Weiss and other mathematicians studied these concepts and established a series of deep results concerning the so called local variational principle of entropy and variational relationship of entropy pairs. However, until now there is still a lot of basic problems in topological K-system remaining open, one of the most essential ones is the structure of this system.

The other important measure-preserving system (MDS) is the measure- theoretical null system introduced by Kushnirenko in 1967, which is the simplest MDS under entropy context. A MDS is null if and only if it has discrete spectrum. Such a system satisfying ergodicity is equivalent to an ergodic rotation on a compact abelian group by the Halmos-Von Neumann theorem. Although the topology counterpart of a measure-theoretical null system was introduced by Goodman in 1974, but a little is known concerning its structure. This is a challenging problem.

Recurrence property is another important concept in dynamics. The study of recurrence properties can be tracked to the classical works of Gottschalk, Hedlund, Furstenberg etc. For example, in 1967 Furstenberg proved that “the return time set of a weak mixing system has filter property”. In topological dynamics (TDS), transitivity, weak mixing and strong mixing are the three basic recurrence properties. There are some recent developments on the topics. In 2000,

Blanchard etc. added a class of recurrent property between transitivity and weak mixing: scattering. Later on, Akin, Glasner and Weiss used the language of family to study the recurrence properties between weak mixing and strong mixing more accurately under a certain sense. So define and classify other natural recurrence properties are problems which scholars in the fields are very concerned of.

In this dissertation, we start from the similarities between MSD and TDS and use the idea of localization (pairs or tuples) to conduct a systematical study of some important problems such as: the relationship between Devaney chaos and Li-Yorke chaos; the structures of topological K-system and topological null system; a finer classification of recurrence properties. There are six chapters in the thesis. The first two are introduction and preliminary knowledge, the latter four are the main results, which can be divided into three parts as follows.

The first part solves the long open problem whether Devaney chaos implies Li-Yorke one. For this purpose, in chapter 3, we classify point pairs of X according to the orbits under the action of T. We discuss the relationship between these specific pairs and chaos. Then for a non-periodic transitive system, we prove their asymptotic relationship and asymptotic class are both of first category. Furthermore, we show that if its proximal relationship is dense in a neighborhood of a point on the diagonal, then the system is Li-Yorke chaotic. As an application, we get that a non-periodic transitive system containing periodic points is Li-Yorke chaotic. Thus, the open problem whether Devaney chaos implies Li-Yorke one is answered affirmatively. At the same time, it is shown that on many compact metric spaces there exist TDS for which the whole space is a chaotic set. Such spaces include some countable compact metric spaces, cantor set and continuum with arbitrary dimension. These results have been published on Ergodic Theory and Dynamical Systems, 21(2001),No.1, 77-91 and Topology and its application, 117(2002), No.3, 259—272. There are a lot of researchers who are interested in these results, including Akin, Auslander, Blanchard, Glasner, Kolyada, Maass, Host, Snoha, Smital, Mai and Jiang. In their papers our results are referenced for 23 times. Specifically, in the paper by Blanchard etc where the authors proved that positive entropy implies Li-Yorke chaos, our results are an important source of references. In the paper of Akin and Kolyada in Nonlinearity, 2003, our results about asymptotic relationships are called ``Huang-Ye equivalence``.

The second part concerns the structure of topological Kolmogorov and null systems. In chapter 4, we study the topological counterpart of a measure-theoretical Kolmogorov system: n-uniformly positive entropy system (n>1) and completely uniform positive entropy system. First, we show that the topological realization of a measure- theoretical Kolmogorov system has completely uniform positive entropy. And we use the measure-theoretical Pinsker algebra to characterize n-entropy tuples, the localized concept of n-uniformly positive entropy. Then we prove a theorem which establishes the relationship of entropy between a measurable cover and a measurable partition, and strengthen the principle of local variation of entropy, from which we get the variational relationship between topological n-entropy tuples and measure-theoretical n-entropy tuples. We also use topological non-trivial partition, interpolated set with positive density, and hyperspace to characterize n-uniformly positive entropy systems and completely uniform positive entropy systems. In the procedure, we solve some open problems proposed by Blanchard, Host, Glasner and Weiss. For example, we prove that n-uniformly positive entropy does not coincide with (n+1)-uniformly positive entropy. Our results provide a general framework for solving such problems and cover almost all the results in the related problems in the past decade. These results will appear in Israel Journal of Mathematics.

In chapter 6, we study the structure of topological null systems. Our idea is to localize the concept of sequence entropy. We define the notion of a sequence entropy pair firstly, then give some sufficient conditions for a pair to be a sequence entropy pair, then we use Ellis semigroup theory to prove that the result of Kushnirenko is true in the topological settings for minimal TDS neglecting an almost one to one extension. These results are published in Ergodic Theory and Dynamical Systems，23(2003), No.5, 1505-1523. At the same time, we use the measure-theoretical Kronecker algebra to prove the set of measure-theoretical sequence entropy pairs is contained in the set of topological sequence entropy pairs. However, the converse is not true, as there is no variational principle for sequence entropy. Using the lifting property of sequence entropy pairs, we show that there exist a maximal null factor and a maximal M-null factor for every TDS. These results will appear in Annales De L'Institut Fourier.

In the third part we introduce some duality properties such as topological mild-mixing, and use these properties to create a finer classification of recurrence properties. In chapter 5, first we introduce the duality concept for a dynamical property stronger than transitivity. Then we define the duality properties for several familiar dynamical properties, such as topological mild-mixing and several types of scattering. And we use the complexity function of an open cover and the recurrent time set to classify the recurrence property. In the procedure, we solve an open problem proposed by Blanchard, Host and Maass at 2000, i.e. we prove that 2-scattering is equalvalent to scattering. We also discover that recurrent time set of several scattering system are important sequences of natural number in the fields of combinatorial number theory, ergodic theory and topological group, such as Poincare sequences and recurrent sequences. Therefore, in the classification of recurrence properties, combinatorial number theory, ergodic theory and topological group are necessary tools. These results are published in Nonlinearity, 15(2002), No.3, 849-862 and Ergodic Theory and Dynamical Systems, 24(2004), No.3, 825-84. Eli Glasner referred to our results in a recent review paper and a series of presentations given at several known universities.

It is worth to mention that the topological mild-mixing defined by us can be viewed as the topological counterpart of the concept of measure-theoretical mild mixing in MDS. Independent of our research, Weiss and Glasner also define the notion. They describe this concept with much content in the chapter “On the interplay between measurable and topological dynamics” of Handbook of Dynamical systems Volume 1B（coming in 2004）edited by B. Hasselblatt and A. Katok. They wrote：“…We note that the definition of topological mild mixing and the above concerning this notion are new. However independently of our work Huang and Ye in a recent work also define a similar notion and give a comprehensive and systematic treatment,…”.

During the PhD study and one year post-doc research, we also obtain the following results concerning the same topic.

1. We partly solve a long open problem proposed by famous mathematician H. Furstenberg in one of his classical papers in 1967: to characterize the systems disjoint from all minimal systems. We show that such a system is weakly mixing with dense minimal points if it is transitive. And we also give a sufficient condition for such a system to be disjoint from all minimal systems. Finally, we show that such a system may not contain periodic points. This result will be published in Transactions of the American Mathematical Society. In a note written by E. Akin he titled it as "Minimal maps: Notes on some results of Huang and Ye".

2. We obtain the relationship between two notions of measure-theoretical entropy of an open cover, and prove that both of them have ergodic decomposition. Furthermore, we get the formula of the local Abramov entropy. These results are published in Ergodic Theory and Dynamical Systems， 24(2004), No.4, 1127-1153.

3. In our papers published on Nonlinearity， 17(2004), No.4, 1245-1260 and Proc. of the Steklov Inst. of Math., Vol 244(2004), 280-287, we discuss the proximal cell of F- mixing system (F is a family) and the minimal sets in an almost equicontinuous system.

Key borad: Complexity, tuples, chaos, entropy, recurrence property