论文题目：黑洞热力学熵和统计学熵

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摘　要

本文分别用砖墙模型、Euclidean 路径积分和 Killing 视界上的共形场论方法研究了四维非极端稳态轴对称黑洞的热力学熵和统计力学熵。

首先， 采用 t' Hooft 提出的把黑洞动力学自由度与量子激发态相对应， 以黑洞外部量子物质场的激发态来研究黑洞统计力学熵的砖墙模型，以及 Pauli-Villars 正规化方案，并把原来的 Dirichlet 边界条件用波函数在视界面附近被散射的边界条件代替，从最一般的四维稳态轴对称黑洞的度规出发，求得了由非最小耦合量子标量场引起的一般稳态轴对称黑洞的统计力学熵的表达式。

而后，通过 Wick 转动由最一般的稳态轴对称黑洞的度规得到具有锥奇性的欧几里德度规，利用 Euclidean 路径积分和热核展开方法，获得了一般稳态轴对称黑洞热力学熵经典部分及由任意耦合量子标量场引起的黑洞熵的一圈修正。计算表明：当取一对与视界面正交且与 1-形式对偶的矢量后，视界外曲率的二次项对熵的贡献项为零。当把洞热力学熵的经典部分与其量子一圈修正进行合并时，我们注意到发散部分可通过引力常数和耦合常数的重整化予以吸收。从而，我们得到了经过重整化的一般稳态轴对称黑洞热力学熵表达式。

由于非最小耦合标量场引起的统计力学熵与正规子相关，我们可通过调节正规子使得一般稳态黑洞的统计力学熵与其热力学熵表达式一致。应该注意的是，在计算统计力学熵和热力学熵时，我们仅采用了一般稳态轴对称时空度规以及该度规所描述的物体是黑洞的必要条件。因此，所得结果不仅适用于已知的稳态轴对称黑洞，而且，当考虑量子场对引力场反作用时，只要反作用不改变时空的对称性，所得到的结果依然成立。

另一方面，我们利用协变相空间技术、Carlip 边界条件、时空的稳态条件、以及稳态时空中电磁场和 Dilaton 场的对称性，分别从具有宇宙项和电磁场的 Lagrange 函数和由弦理论得到的四维低能等效理论的 Lagrange 函数出发，得到了一般稳态轴对称荷电黑洞、以及静态和稳态 Dilaton 黑洞时空中的约束代数。作为例子，我们考虑了 Kerr-Newman 黑洞、Kerr-Newman-AdS 黑洞，静态 Garfinkle-Horowitz-Strominger Dilaton 黑洞、Garfinkle-Maeda Dilaton 黑洞、以及稳态 Kaluza-Klein 黑洞。通过引入相应的单参数微分同胚群元，我们在这些黑洞的 Killing 视界面上构造出具有中心荷的 Virasoro 子代数。用标准共形场论方法由中心荷求得了态密度，从而得到了这些黑洞的统计力学熵。它们都与其相应的 Bekenstein-Hawking 熵一致。因此，所得结果为黑洞熵的统计力学起源研究提供了重要线索。

在共形场论方法中，我们还计算了黑洞统计力学熵的量子修正。我们注意到，为了使得从不含修正的 Cardy 公式得到的黑洞统计力学熵与黑洞 Bekenstein-Hawking 熵一致，我们必须把

周期 T 取为 Euclidean 黑洞的周期。于是，所得的 Virasoro 子代数的中心荷与黑洞视界面积成正比。这一约束条件表明我们不能象 Carlip 所猜想的那样通过调节周期 T 使 Virasoro 代数的中心荷变成与黑洞面积无关的量。因此，利用含量子修正的 Cardy 公式我们得到：黑洞统计力学熵的一级量子修正与黑洞 Bekenstein-Hawking 熵的对数成正比，比例系数是负二分之一。由此我们发现，黑洞具有比 Bekenstein 熵界更严的新熵界。

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关键词：黑洞，熵，量子场论，共形场论，热力学，统计力学。

ABSTRACT

Both statistical-mechanical and thermodynamical entropies of the nonextreme stationary axisymmetric black holes in four-dimensional spacetime are investigated by ``Brick wall" model, Euclidean path integral method and conformal field theory at Killing horizon, respectively.

First, by using the 't Hooft's ”Brick wall“ model in which the black hole degrees of freedom are identified with the ones of a quantum gas of particles and the statistical-mechanical entropy is arisen from a thermal bath of quantum fields propagating outside the horizon, and replacing the Dirichlet condition by scattering ansatz for the field functions at the horizon and with Pauli-Villars regularization scheme, an expression for the statistical-mechanical entropy of the general stationary axisymmetric black hole arising from the nonminimally coupled scalar fields is obtained.

Then, we showed that an Euclidean manifold, which is obtained by Wick rotation of the general stationary axisymmetric geometry, has a conical singularity. By means of the Euclidean path integral method and heat kernel approach, we obtained tree-level thermodynamical entropy and one-loop quantum corrections for the general stationary axisymmetric black hole due to a nonminimally coupled scalar field. We prove that the contribution of the quadratic combinations of the extrinsic curvature of the horizon is zero by defining a pair of vectors which orthogonal to the event horizon and dual to one forms. By combing the tree-level entropy with one-loop correction, we find that the divergence can be absorbed in the renormalization of the gravitational and coupling constants. After being renormalized with the standard scheme, the thermodynamical entropy of the general stationary axisymmetric black hole is found.

By adjusting the regulators, we can set the statistical-mechanical entropy of the general stationary axisymmetric black hole equal to the thermodynamical entropy since the statistical-mechanical entropy is related to the regulators. We should note that we only use the most general metric for the stationary axisymmetric spacetime and the conditions that the object described by the metric is a black hole in calculations of the statistical-mechanical and thermodynamic entropy. Therefore, the results are valid not only for stationary axisymmetric black holes that we have known, but also for the case that the quantum field possesses the back reaction to the gravitational field so long as the back reaction does not affect the symmetry of the spacetime.

On the other hand, with Carlip's boundary conditions and symmetries of the Maxwell and Dilaton field in the general stationary axisymmetric spacetime, the constraint algebras of the stationary axisymmetric charged black holes and stationary Dilaton black holes are constructed from Lagrangian with a cosmological term and electromagnetic fields and the low-energy effective field theory describing string by using covariant phase technique. As examples, the standard Virasoro subalgebras with corresponding central charge for the Kerr-Newman black hole, Kerr-Newman-AdS black hole, the static Garfinkle-Horowitz- Strominger dilaton black hole, the Gibbons-Maeda dilaton black hole, and the stationary Kaluza-Klein black hole are obtained by introducing one-parameter group of diffeomorphism at Killing horizon. The statistical-mechanical entropies of these black holes yielded by standard conformal field theory methods agree with their Bekenstein-Hawking entropies. Therefore, the results present an important clue for the statistical explanation of the black hole entropy.

We also consider first-order quantum correction to the statistical-mechanical entropy in the conformal field theory. We know that the central charge of the Virasoro subalgebra is proportional to area of the horizon since we have to take T as periodicity of the Euclidean black hole in order that the statistical-mechanical entropies of the black holes yielded by standard Cardy formula agree with their Bekenstein-Hawking entropies. It is show that we can not do what as Carlip suggested that the central charge can be changed into a constant in a sense of being independent of the horizon area by adjusting the periodicity. Making using of the new Cardy formula we find that the quantum correction to the entropy contains a logarithmic term with a factor –1/2. The result shows that we have an new entropy bound which is tighter than the standard holographic entropy bound due to Bekenstein.

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Keywords： Black Hole， Entropy， Quantum Field Theory， Conformal Field Theory， Thermodynamics， statistical Mechanics.