论文题目:离散竞争动力系统的一般性质及反应扩散方程解的收敛性

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摘　要

竞争系统的研究具有很长的历史，它诞生于上一世纪 20 年代，由意大利数学家 V. Volterra 所开创。在上世纪 80 年代后，著名数学家 S. Smale 和 M. W. Hirsch 开创了这一分支研究新阶段------利用动力系统原理研究竞争系统共有的动力学性质。而后 M. W. Hirsch 和 H. Matano 创造的单调动力系统理论又迅速促进了竞争系统的发展。近年来，E. C. Zeeman，M. L. Zeeman 以及由 ICM98 一小时报告者 K. Sigmund 为首研究小组对竞争 Lotka-Volterra 系统的研究工作尤为突出。

以上的工作都集中于自治系统，而当考虑到环境条件因时间或季节而周期变化时，就相应产生了竞争周期系统，利用周期系统的 Poincaré映射往往可将周期系统解的动力学性态用离散动力系统来刻划。而对离散竞争系统的研究吸引了包括 J. K. Hale, P. Hess, H. L. Smith 以及 P. Takac 在内的大量学者的注意。

其中，H. L. Smith 率先对反映种群生态的周期竞争 Kolmogorov 系统产生的 Kolmogorov 型离散竞争动力系统进行了深入讨论，并留下了搁置近 20 年之久的关于离散竞争动力学的“Smith”猜测：即，若每个物种都能在各自的唯一不动点上生存（原点为排斥子），则在大范围意义下，原点排斥子的上边界Σ（称为负载单形）是系统的全局吸引子，且Σ是无序的并通过径向投影同胚于标准 n-1 维概率单形。围绕这一猜测还留下如：负载单形Σ的光滑性、周期轨非稳定流形的位置等一系列相关的重要公开问题。

“Smith”猜测暗示了 S.Smale 的关于自治竞争 Kolmogorov 系统的构造绝非偶然。 同时也显示了研究三维 Kolmogorov 型竞争周期系统就象研究两维周期系统一样，从而可以为研究诸如三维 Kolmogorov 型周期竞争系统无穷多个次调和解的存在性、负载单形Σ内具有 Hamiltonian 结构的条件等许多重要问题提供理论基石。

1988 年，M. W. Hirsch 利用单调自治系统的极限集两分法理论，证明了这一猜测在自治情形下成立。遗憾的是，M. W. Hirsch 证明的核心理论---极限集两分法只适用于自治系统而在离散系统中却不成立，要解决该猜测必须采用新方法。因此该猜测及其相关问题一直未被解决并成为离散竞争动力系统中的重要难题。另外关于负载单形Σ的光滑性问题即使在自治情形下也仍被 M. W. Hirsch 提为公开问题。

本篇论文的第一部分将致力于解决“Smith”猜测及其相关公开问题。
为解决“Smith”猜测，我们首先系统地研究了强序拓扑向量空间上抽象离散竞争动力系统的一般性质。对于在强序拓扑向量空间上的抽象离散竞争动力系统，我们研究了极限集的序

结构和极限集的几何位置: 利用非振动原理及和全新的 -不变超曲面理论证明了任何α-或ω-极限集是无序的且位于一个完全无序、余维为 1 的不变 Lipschitz 子流形上。这精确说明了离散竞争动力系统的动力学性态本质是余维 1 的，从而把 M. W. Hirsch 关于竞争连续流动力学性态的结果推广到非常一般的状态空间上的离散竞争动力系统中。同时，这一结果为"Smith"猜测的解决作了重要铺垫。作为这一结果的另一重要应用，我们还首次证明了关于平面竞争映射的 Sarkovskii 定理。我们还进一步讨论了平面竞争映射产生的 Li-York 浑沌等复杂动力学性态。

随后，我们利用已得到的结果解决"Smith"猜测及其相关公开问题。以下为基本步骤：对于抽象的有限维 Kolmogorov 型离散竞争系统，

(1)我们首先在耗散性及锥内部强竞争性的假设下证明了存在一簇至多可

数个互不相交的 n-1 维不变子集，它们吸引所有不渐近于周期轨的持续轨道；作者同时还深刻分析了这些 n-1 维不变子集的拓扑结构和相互联系。

(2)在此基础上，我们利用全新的 -不变超曲面理论和上（下）极限集的

ω-稳定性理论彻底解决了"Smith"猜测。并且证明了任意周期轨的非稳定流形都位于负载单形Σ上。

(3)紧接着，我们利用向量丛指数分离理论、Oseledec 遍历理论、Pesin 理

论及 persistent 理论解决全局吸引子Σ的 光滑性。我们得到了若系统是 的且限制到每一个子面上都是 uniformly persistent 的，则其全局吸引子Σ是 的。 从而使得 Hirsch 提出的关于连续竞争动力系统负载单形光滑性公开问题成为本文结果的自然推论。

在本论文的第二部分，我们将研究有序 Banach 积空间上单调动力系统的收敛性及其在（周期）拟单调反应扩散方程组初边值问题中的解渐近性态的应用。

有关此方面现有的几乎所有结果都是建立在单个偏微分方程或不可约拟单调反应扩散方程组上的，之所以如此是因为这两类方程的解在适当的 Sobolev 或分数幂空间上具有强单调性质。包括 Matano, Dancer 及 Smith 在内的数学家指出在没有不可约(强单调)的假设下解决收敛性是困难的。但遗憾的是，在模拟生物生态系统时往往反应扩散系统却总是非强单调的, 例如 Lotka-Volterra 反应扩散系统等等。为了解决这一问题，我们创造了一种全新的方法，在没有不可约(强单调)的假设下，利用有序 Banach 积空间中单调系统的抽象理论统一处理了子齐次、弱子齐次以及拥有保序首次积分等几类周期(自治)拟单调反应扩散方程组非负解的渐近周期(收敛)性问题。具体地说，我们将极限集与特殊的不动点作比较，从而在无限维系统中构造出一个有限整数值函数 ，利用该整数值函数的递归性质并结合部分度量理论以及非线性泛函分析中的正算子理论对原有序 Banach 积空间的坐标子空间的个数进行归纳从而最后得出系统的收敛性。同时我们把这些结果应用于模拟生物生态系统的 Lotka--Volterra 反应扩散方程组， 从而在最合理的弱假设下推广了包括数学家 Hirsch，Dancer，Hess 和 Takac 等许多前人的结果。同时我们的结果对模拟生物生态系统的反应扩散方程组的应用解决了无法将强单调理论应用于此类方程组的棘手问题。

Abstract

The earliest investigation of competitive systems can be traced back to 1920's, due to the work by Italian Mathematician V. Volterra. After the mid-1980s, the research on competitive systems, beginning with the path-outbreak contributions by S. Smale and M. W. Hirsch, have reached a new stage, that is, the theory of dynamical systems is introduced in studying the common properties of competitive systems. Thereafter, competitive systems have undergone extensive investigations due to the monotone dynamical systems developed by M W. Hirsch and H. Matano. Recently, important progress in competitive Lotka-Volterra systems has been presented by E. C. Zeeman, M. L. Zeeman and the research group leaded by K. Sigmund, who gave a plenary lecture on ICM 98.

The above-mentioned work focuses on autonomous systems. If one wants to model systems with day-night and seasonal variant, he needs to study competitive systems with time periodic. One can describe the asymptotic behavior of the solutions according to the discrete-time dynamics of the Poincare mapping associated with the periodic systems. Many mathematicians have involved in the investigations of discrete-time competitive systems, such as J. K. Hale, P. Hess, H.L. Smith and P. Takac, etc.

The dynamics of Kolmogorov-type competitive mapping associated with the time-periodic competitive Kolmogorov systems of ODEs were first studied by H. L. Smith, who Conjectured that if each species could survive in the absence of the others at a unique fixed point (the origin is a repeller) then the boundary Σ of the basin of repulsion of the origin should contain the global attractor for the dynamics. Σ(called carrying simplex) is unordered and homeomorphic to the standard probability (n-1)-simplex by radial projection. Related to Smith Conjecture, there are several important open problems remaining, such as the smoothness of the carrying simplex Σ, the position of the unstable manifolds of the periodic orbits, etc.

Smith Conjecture implies the essence of Smale's construction in autonomous competitive systems. Furthermore, it shows that the asymptotic behavior of 3-dim periodic competitive Kolmogorov systems is the same as that of planar periodic systems, with which one can investigate the important problems such as, the existence of infinitely many subharmonic solutions in 3-dim periodic competitive Kolmogorov systems, the Hamiltonain structures in the carrying simplex Σ,etc.

M. W. Hirsch (1988) proved the Conjecture in the autonomous case by the approaches of limit-set dichotomy in autonomous monotone dynamical systems. Unfortunately, the limit-set dichotomy DOES NOT hold in the discrete-time cases. It needs new methods to solve this Conjecture. Therefore, Smith Conjecture and its related open problems in discrete-time case have had little progress for almost 20 years and become one of the most difficult problems in discrete-time competitive dynamical systems. Moveover, Hirsch posed the open problem of the smoothness of the carrying simplex even in autonomous cases.

The purpose of the first part of this thesis is devoted to prove Smith Conjecture and its related open problems.

Firstly, for competitive discrete-time dynamical systems on a strongly ordered topological vector space, we prove that any limit set is unordered and lies on some

invariant hypersurface with codimension one, which generalizes M.W. Hirsch's

results for competitive autonomous systems of ordinary differential equations to competitive maps in a very general framework and implies the Sarkovskii's Theorem for planar strongly competitive maps. We also offer some criteria for planar competitive maps having Li-York chaos. Furthermore, this result plays an important role in solving Smith Conjecture.

Secondly, we focus on Smith Conjecture and its related problems. The steps are as follows:

(1) Under the hypotheses of dissipation and strong competition, we proved that there

is a canonically defined countable family of disjoint, invariant sets which attract all persistent trajectories whose limit sets are not cycles.

(2) We utilized the new theory of the invariant d-hypersuface and the ω-stalility of

the upper and lower limit sets to solve Smith conjecture, and prove that the unstable manifold of any m-periodic point is contained in Σ.

(3) Based on the vector bundle exponential separation theory, Oseledec ergodic

theorem, Pesin theory and Persistent theory, we prove that the carrying simplex Σ is of provided the system is of and restricted on any face is uniformly persistent. Thus the open problem posed by Hirsch has been a corollary of our result.

In the second part of this thesis, we investigate the convergence in monotone discrete dynamical systems on product Banach spaces and its applications to the (periodic) quasimonotone reaction-diffusion systems.

As far as the author knows, almost all the results on this topic are on the one PDE equations or the system of PDEs with irreducible quasimonotone hypothesis, from which one can induce a strongly monotone system on some Sobolev space or fractal power space. Many mathematicians, such as Matano, Dancer and Smith, mentioned that it is a difficult question to prove the convergence without irreducible assumption. On the other hand, the models from ecosystems and ecology are almost reducible. In order to solve this problem, we create a new method in comparing the limit set with some special fixed point (equilibrium) and construct an integer-valued function M(u,p). Using such a function, we first prove the convergence to fixed point for subhomogeneous and monotone discrete-time dynamical system, and then the

convergence to steady state of quasimonotone reaction-diffusion systems, which are either subhomogeneous, or weakly-subhomogeneous or with an first integral, without the irrducibility hypothesis. These results generalize lots of previous results by Hirsch，Dancer，Hess and Takac. We also apply these results to Lotka-Volterra reaction-diffusion systems.